ON THE ORIGIN OF CONVENTION:
EVIDENCE FROM COORDINATION GAMES

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Abstract: We report the results of a coordination game experiment. The experiment carefully distinguishes between conventions based on labels and conventions based on populations. Our labels treatments investigate the abstraction assumptions that underlie the concept of a strategy, while our population treatments investigate the attraction of alternative mutually consistent ways to play under adaptive behavior. We observe conventions emerging in communities with one population and labels and with two populations and no labels, but the most effective treatment is two labeled populations.

A final section investigates individual subject behavior. Specifically, we estimate logistic response learning models. Of the models considered, a version of exponential fictitious play fits our data best.

Key words: convention, labels, populations, coordination, dynamical systems, adaptive learning, exponential fictitious play, human behavior.

JEL classification: c72, c78, c92, d83.

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I. INTRODUCTION

Two related problems exist in the theory of equilibrium points. First, the mutual consistency requirement of an equilibrium assignment is not an implication of individual rationality, because individual rationality does not restrict the subjective beliefs a player may hold. Rather, individual rationality means internal consistency and internally consistent beliefs and actions of different players may not be mutually consistent. Second, there is often more than one mutually consistent strategy combination, which results in an indeterminant analysis. Consequently, understanding the origin of mutually consistent behavior is an essential complement to the theory of equilibrium points.[1]

It is possible to construct a deductive equilibrium selection theory by introducing abstraction assumptions that go beyond individual rationality and mutual consistency: assumptions like efficiency, symmetry, and security. Deductive selection principles select equilibrium points based on thinking about the description of the game. However, if the deductive approach is to provide an accurate theory of observable games, the selection principles of efficiency, symmetry, and security must formalize characteristics that are commonly known to be psychologically salient.

Van Huyck, Battalio, and Beil (1990; 1991) and Van Huyck, Cook, and Battalio (forthcoming) present evidence against the psychological salience of efficiency.[2] Van Huyck *et al.* (1995) present evidence against the psychological salience of symmetry in symmetric bargaining games.[3] In these experiments, security undermines the salience of either efficiency or symmetry.[4] These facts make us pessimistic about the usefulness of constructing purely deductive selection theories.

Repeated interaction may allow players to learn to coordinate on a mutual best response outcome. Following Lewis (1969), we distinguish between historical precedents in repeated games and conventions in evolutionary

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[1] See Kreps (1990, ch.6) for a non-technical discussion of the basic issues. See Bernheim (1984) and Brandenburger (1992) on the mutual consistency requirement. See Harsanyi/Selten (1988) on equilibrium selection.

[2] See also Cooper, DeJong, Forsythe, and Ross (1990; 1994) and Friedman (1996).

[3] Roth/Schoumaker (1983) and Binmore, *et al.* (1993) present evidence against equal-division in asymmetric Nash demand games.

[4] Of course, they don't claim that security always undermines efficiency and symmetry. Straub (1995) found that risk-dominance makes more accurate predictions than security in the class of coordination games he considers.

games. Selecting a mutual best response outcome based on precedent requires actors to focus on some salient analogy to a shared past instance of the present observable game and to expect others to focus on the same analogy. Crawford and Haller (1990) explain how players should use precedent to establish a common language that will allow players to resolve strategic uncertainty in repeated coordination games. Meyers *et al*. (1992) found that if subjects do not use precedent optimally in early periods of the repeated game it was very difficult for them to learn to do so.

Convention generalizes precedent to situations where one lacks shared experience, but knows that everyone involved is a member of the same community. An observable regularity in the behavior of members of a community in a recurrent situation is a *convention* if it is customary, expected, and mutually consistent, compare Lewis (1969) and Young (1993).

As mentioned above, Van Huyck, *et al*. (1995) found that security can undermine the salience of symmetry. Initially, this resulted in outcomes that were not mutually consistent. However, in one treatment the matching protocol divided subjects into two labeled populations: a row population and a column population. In five of eight sessions under this treatment they observed unequal-division conventions emerging in communities of symmetrically endowed subjects. In that paper, no distinction was made between conventions based on labels, the row and column position in the game, and conventions based on populations, own population players only meet other population players

This distinction reflects two different sources of mutually consistent behavior. Labels may serve as a focal point that solves the strategy coordination problem if their significance is recognized by members of the community, see Sugden (1995) for references. Alternatively, changing the matching protocol from one to two populations changes the state space of models of population dynamics and for many population dynamics this change has the implication that only strict equilibria are asymptotically stable, see Weibull (1995) for references. Consequently, inefficient but symmetric mixed strategy equilibria are no longer asymptotically stable.

This paper attempts to seperate the influence of labels and populations on the ability of subjects in an evolutionary coordination game to adopt a conventional way to play. Our labels treatments investigates the abstraction assumptions that underlie the concept of a strategy, while our population treatments investigate the attraction of alternative mutually consistent ways to play under adaptive behavior. We observe conventions emerging in communities with one population and labels and with two populations and no labels, but the most effective treatment is two labeled populations.

A final section, added in response to helpful comments by several referees, investigates individual subject behavior. Specifically, we estimate logistic response learning models, like Mookerjee and Sopher (1994) and Cheung and Friedman (1995). Of the models considered, a version of exponential fictitious play fits our data best. When estimated by treatment, the exponential fictitious play models converge toward best response fictitious play as one moves from the one population no labels treatment to the two labeled population treatment.

II. Analytical Framework

In order to focus the analysis, we use the following generic coordination game. Two players are matched and asked to choose either a 1 or 2. If they choose different numbers, they each earn 40 cents. If they choose the same number, they earn nothing.

Table 1 reports the earnings tables used in the experiment. The main difference between the two earnings tables is how they are labeled. In the no labels treatment, the rows are labeled "your choice" and the columns are labeled "other participant's choice". In the labels treatment, subjects were labeled either row or column and the earnings table described their potential earnings according to "row choice" and "column choice". We do not mean anything deeper by the terms 'no labels' and 'with labels' than this difference in the experimental design.

No Labels

Earnings Table

		Other Participant's Choice 1	2
Your Choice	1	0	40
	2	40	0

Labels

Earnings Table

		Column Choice 1	2
Row Choice	1	0,0	40,40
	2	40,40	0,0

Table 1: Earnings table for no labels and labels treatments.

Making the usual abstraction assumptions gives a 2×2 game with three Nash equilibria. Let p_i denote the probability that player one chooses action i, where $i \in \{1,2\}$ and let p denote the vector (p_1,p_2). Let q_i denote the probability that player two chooses action i, where $i \in \{1,2\}$ and let q denote the vector (q_1,q_2). Player one's strategy vector p is an element of the simplex $\mathbf{S}^2 = \{x \in \mathbb{R}^2 | \, x_i \geq 0, \Sigma x_i = 1, i = 1,2\}$; as is player two's strategy vector, $q \in \mathbf{S}^2$. A strategy combination is the 4-tuple $\{p,q\}$. The unit coordinate vectors $e_1 \equiv (1,0)$ and $e_2 \equiv (0,1)$ denote pure strategies: the actions 1 and 2 respectively. In this notation, the Nash equilibria are $\{e_1,e_2\}$, $\{e_2,e_1\}$, and $\{(½,½),(½,½)\}$.

Player one's expected payoff is $p.G.q$ and player two's expected payoff is $q.G.p$, where G is the payoff matrix corresponding to the earnings tables given in table 1. Game G is defined by the following description: $\mathbf{G} \equiv \langle\{1,2\}, \{p.G.q,\ q.G.p\}, \{\mathbf{S}^2,\mathbf{S}^2\}\rangle$.[5] Game G is symmetric in that the expected payoff functions are symmetric and the players' feasible strategies are the same.

Harsanyi/Selten (1988;p.73) argue that symmetry is an “indispensable requirement for any rational theory of equilibrium point selection that is based on strategic considerations exclusively.” A strategic analysis of game G that requires symmetry selects the mixed strategy equilibrium. However, this equilibrium is inefficient. The expected payoff is half that of either of the asymmetric pure strategy equilibria. In game G, symmetry conflicts with efficiency. Restricting attention to the information in **G** has the advantage that it allows a general analysis and the disadvantage that it selects an inefficient equilibrium assignment.

Up to this point we have ignored the differences in how the earnings tables are labeled. Doing so is consistent with an analysis that is invariant with respect to renaming players. Suppose nature randomly assigns the labels “row” and “column” to player 1 and 2 and it is commonly understood that players condition their actions on their labels. Let σ_{ij} denote the pure strategy “play *i* if labeled row and *j* if labeled column.” The strategic form of this two stage game would no longer be 2×2. The payoff bi-matrix is given in table 2.

	σ_{11}	σ_{12}	σ_{21}	σ_{22}
σ_{11}	0,0	20,20	20,20	40,40
σ_{12}	20,20	40,40	0,0	20,20
σ_{21}	20,20	0,0	40,40	20,20
σ_{22}	40,40	20,20	20,20	0,0

Table 2: Payoff bi-matrix when strategies are contingent on labels: Game Γ.

Let Γ denote the 4×4 game represented in table 2. It has four strict equilibria and an infinite number of mixed strategy equilibria. However, two

[5] This game is sometimes called an intersection game, see for example Lewis (1969,p.6), Sugden (1986, Ch.3), Crawford (1991, p.36-38), or Samuelson (1991, p.114).

of the strict equilibria are both efficient and symmetric. Using notation analogous to that used in the 2×2 game, the efficient symmetric equilibria are $\{e_2,e_2\}$ and $\{e_3,e_3\}$. Either both "play 1 when labeled row and 2 when labeled column" or "play 2 when labeled row and 1 when labeled column". Labeling has not solved the basic strategy coordination problem, since there are two efficient symmetric equilibria. Instead it forces one to consider how meaning becomes attached to a 'strategically' irrelevant detail of the game.

III. Evolutionary Games and Replicator Dynamics

Repeated interaction amongst members of a community may allow a convention to emerge that solves their strategy coordination problem. In an evolutionary game, a constituent game S is played by n actors randomly drawn from a community C. In general, C will consist of heterogenous populations, where this heterogeneity may arise either from strategic asymmetries in S or from non-strategic asymmetries in the matching protocol. Even when S is played by strangers, the knowledge that they are members of C may allow them to coordinate on a mutual best response outcome, because members of C conform to a convention.

How do conventions evolve? An evolutionary analysis focuses on the distribution of actions in populations of anonymously interacting players.[6] Here we consider two random pairwise matching protocols either of which could lead to the strategic situation analyzed in section II. In the one population protocol, players are matched with members of their own population. In the two population protocol, players are matched with members of a different population.[7] The protocol describing how players are matched plays an important part in determining the dimension of the resulting dynamical system and the stability of the dynamical system's fixed points.

Let s_i^k denote the fraction of population k using action i and let s^k denote the vector of all s_i^k. All feasible population frequency vectors s^k lie on the simplex, either $\mathbf{S}^2$ without labels or $\mathbf{S}^4$ with labels. (Notice that this is the same space as an individual player's strategy space.) The state space equals $\mathbf{S}^2$ in the one population no labels case, $\mathbf{S}^4$ in the one population with labels case, $\mathbf{S}^2 \times \mathbf{S}^2$ in the two population no labels case, and $\mathbf{S}^4 \times \mathbf{S}^4$ in the two

[6] See Sugden (1986), Van Damme (1987), Crawford (1991), Friedman (1991), and Samuelson and Zhang (1992).

[7] We will continue to use the word symmetric in the strategic sense of symmetric payoff functions and identical strategy spaces. Biologists call the one population protocol a symmetric contest and the two population protocol an asymmetric contest.

population with labels case. Let s denote an element of the state space.

A model of adaptive behavior specifies how a state s evolves through time. Let $s(t)$ denote the state at time t. An admissible dynamical system, $ds/dt = F(s)$, generates a unique solution curve $s(t)$ from given initial conditions $s(0) \in \mathbf{S}$. $F(\bullet)$ is itself derived from a payoff matrix, assumptions about adaptive behavior, and the matching protocol. A state s is a *fixed point* of $F(\bullet)$ if all components of $F(s)$ are 0. The state s is a fixed point in the sense that if $s(0)$ is a fixed point of $F(\bullet)$, then $F(s(0)) = \mathbf{0}$ and $s(t) = s(0)$ for all $t > 0$. A state s^* is a *stable fixed point* of $F(\bullet)$ if $F(s^*) = \mathbf{0}$ and it has an open neighborhood $\mathbf{N} \subset \mathbf{S}$ such that $s(t) \to s^*$ as $t \to \infty$ whenever $s(0) \in \mathbf{N}$.[8] The union of all solution curves that tend toward s^* as $t \to \infty$ is called the *basin of attraction* of s^* and is denoted by $\mathbf{B}(s^*)$.

A dynamical system provides a theory of the origin of convention in the following sense: Given an initial state $s(0)$ with solution curve $s(t) \in \mathbf{B}(s^*)$, the dynamic predicts that after a transition period every state will stay so close to s^* as to be indistinguishable from it. When s^* is consistent with a strict equilibrium of the related game, the dynamic predicts the emergence of the specific convention s^*. Hence, the theory predicts whether and, if so, which convention will emerge using information on the games description, the matching protocol, the initial state $s(0)$, and assumptions about adaptive behavior.

Replicator dynamics arise if the growth rate of a behavior in a population is equal to its relative "fitness."[9] Assume that the "fitness" of an action is equal to its expected payoff in the current state. Let A denote a payoff matrix. The expected payoff for a player using action i with one population is $e_i.A.s$. The average expected payoff in state s is $s.A.s$. Hence, the replicator dynamic for payoff matrix A with one population is given by the following system of non-linear differential equations:

$$\dot{s}_i = s_i(e_i.A.s - s.A.s), \qquad \forall i.$$

We are interested in finding the stable fixed points of this system and the basin of attraction of the stable fixed points. From the biology literature, we know that the stable fixed points of the replicator dynamic are a subset of the Nash equilibria of the related game A, see Hofbauer and Sigmund (1988). Rather

[8] The stability concept used in this paper is asymptotic stability, see Hirsch and Smale (1974).

[9] See Borgers/Sarin (1993) for analysis of learning models that give rise to replicator dynamics. See Weibull (1995) for a model of imitation that gives rise to replicator dynamics.

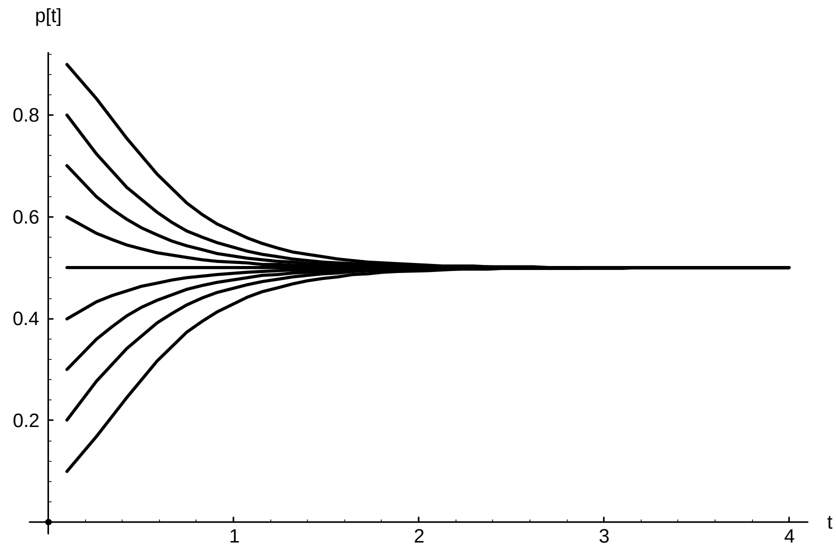

Figure 1: Solution paths of replicator dynamic for game G with one population and no labels, where p[t] is the proportion of the population playing action 1 at time t.

than attempting to derive closed form solutions for the dynamical systems considered in this paper, we will rely mainly on phase portrait methods and numerical analysis: specifically, the Runge-Kutta method described in Maeder (1990, p.172).

Figure 1 graphs the solution paths of the one population replicator dynamic for game G. The dynamic converges to the globally stable fixed point (½,½), which corresponds to the mixed strategy equilibrium of G, for all interior initial states. Hence, the replicator dynamic selects the inefficient but symmetric equilibrium without multiple populations or labels.

Figure 2 graphs the phase diagram of the replicator dynamic for game Γ with one population and labels. The 4 dimensional state space can be represented by a three dimensional tetrahedron, where a point is the vector (s_1,s_2,s_3), using the restriction that $s_4 = 1 - s_1 + s_2 + s_3$ to infer the frequency of strategy σ_{22}. The figure is a two dimensional representation of the tetrahedron. It is being viewed from the point (1,1,1) and the hidden vertex is (0,0,0), which corresponds to the state where all members of the population use strategy σ_{22}.

The stable fixed points of the one population replicator dynamic for game Γ are (0,1,0,0) and (0,0,1,0), which corresponds to the two symmetric efficient equilibria of game Γ. The dynamic divides the tetrahedron into two equal sized basins of attraction separated by a plane connecting the points {(1,0,0), (0,0,0), (0,½,½)}. Hence, the dynamic predicts that the convention that will emerge in the evolutionary game will depend on the historical accident of the initial state. Specifically, observing the initial state allows one to predict

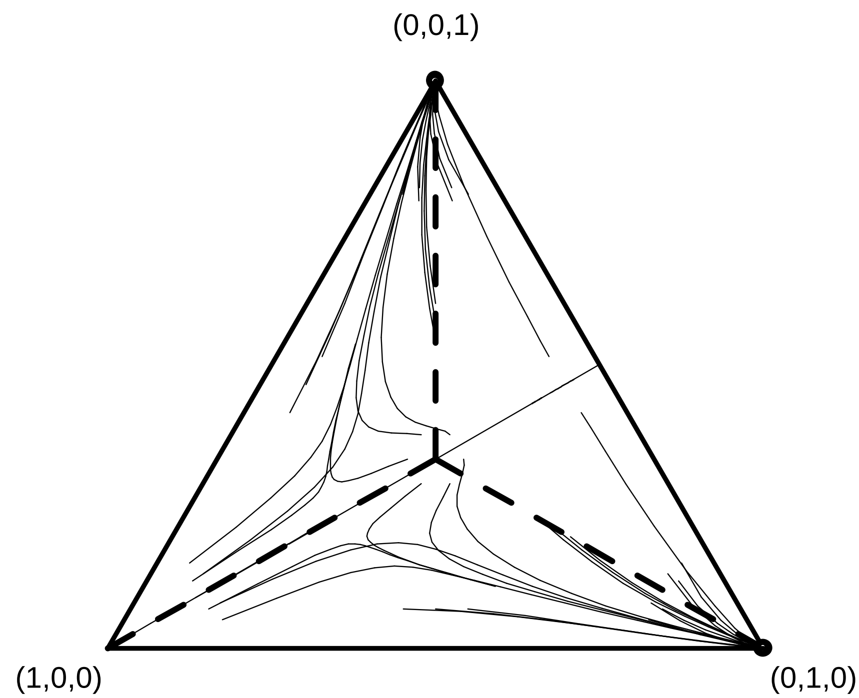

Figure 2: Simulated paths of the one population replicator dynamic for game Γ.

whether the convention will be "play 1 when labeled row and 2 when labeled column" or "play 2 when labeled row and 1 when labeled column."

Notice also that the mixed strategy (¼,¼,¼,¼), which conditional on a player's label is equivalent to the equilibrium mixed strategy of game G, is mutually consistent, but it corresponds to an unstable fixed point of the dynamic. The dynamic predicts that ignoring one's label is not stable. In other words, members of the community will learn to use their labels to solve the strategy coordination problem.

In the two population case, the expected payoff to a player choosing an action depends on his membership in either population. In the experiment these populations were refered to as "your" and "other". For notational convenience index the populations with 1 and 2. The expected payoff to action i for a member of population 1 is $e_i.A.s^2$, while the expected payoff to action i for a member of population 2 is $e_i.A.s^1$. In state s, the average expected payoff for population 1 is $s^1.A.s^2$ and for population 2 is $s^2.A.s^1$. Hence, the replicator dynamic for payoff matrix A with two populations is given by the following system of differential equations:

$$\dot{s}_i^1 = s_i^1(e_i.A.s^2 - s^1.A.s^2), \qquad \forall i,$$
$$\dot{s}_i^2 = s_i^2(e_i.A.s^1 - s^2.A.s^1), \qquad \forall i.$$

Figure 3 graphs the phase diagram for the two population replicator dynamic for game G. The 45° line divides the state space into two basins of attraction. The fixed point $\{(½,½),(½,½)\}$, which was stable under one population, lies on the separatrix and, hence, is now unstable. The stable fixed points are $\{e_1,e_2\}$ and $\{e_2,e_1\}$, which correspond to the strict equilibria of game G. The dynamic converges to a state in which all members of population 1 adopt one action and all of the members of population 2 adopt the other action. Notice that this solution does not require labels or even that players are aware of how the matching protocol allows them to solve the strategy coordination problem.

The shaded regions of figure 3 denote that region of a stable fixed point's basin of attraction in which, given what others are doing, all players have an incentive to conform to the convention. When a state is contained within this region we will say that a convention has emerged, just as one says that it is an American convention to drive on the right side of the road even thought everyone knows that there are drunk drivers on the road. As Lewis (1969) argued, it is useful and important to distinguish between degrees of conformity to a convention. Here we will measure conformity by the minimum with respect to labels or populations of the percent of actions conforming to the convention.

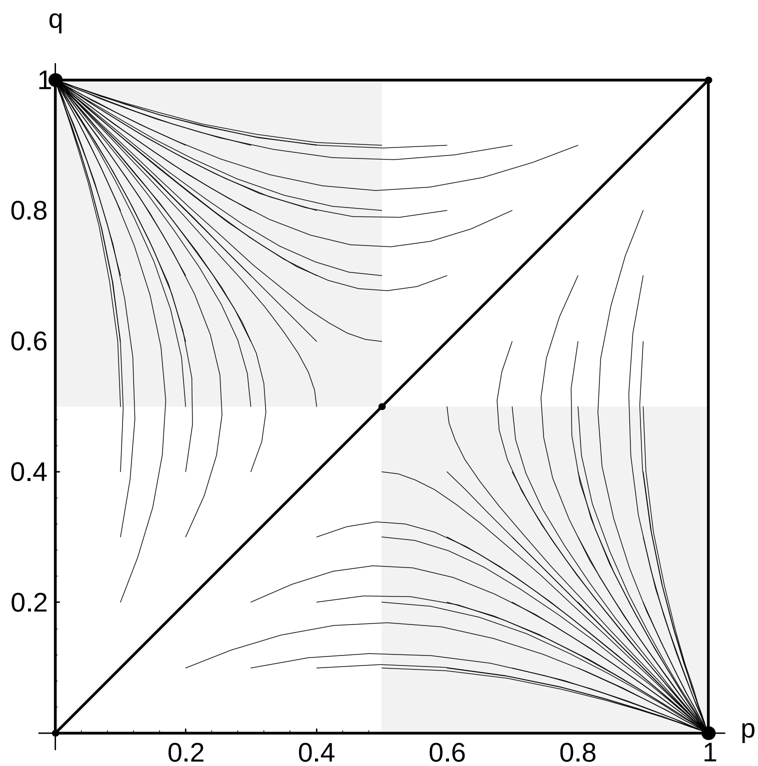

Figure 3: Phase diagram of replicator dynamic for game G with two populations and no labels. p denotes the proportion of population 1 playing action 1 and q denotes the proportion of population 2 playing action 1.

The two population replicator dynamic for game Γ has eight dimensions and is messy to analyze. Fortunately, labels and populations were perfectly correlated in the experiment and imposing this condition collapses the system to that depicted in figure 3. We will refer to this treatment as two labeled populations.

III. Experimental Design

An essential feature of the above analysis is the ability to alter the state space of the dynamic either by having players condition on labels or by altering the matching protocol. As emphasized by Bacharach (1993) and Sugden (1995), the strategy space in an observable game depends on how the players describe the situation to themselves. For example, it may or may not occur to a subject to condition their action on their label. A crucial design issue is whether to force subjects to report strategies or actions. We thought it would be more interesting to allow subjects to discover the map between labels and actions on their own. Hence, we only elicited actions. Humansubjectsplayed the generic coordination game either with or without labels and under a one or two population protocol. Eight subjects participated in each one population session and fourteen subjects participated in each two population session: seven subjects in each population. Table 3 summarizes the experimental design.

Session	Labels	Populations	Number of Periods	Number of Subjects
1	No	One	75	8
2	No	One	75	8
3	No	One	75	8
4	Yes	One	75	8
5	Yes	One	45	8
6	Yes	One	45	8
7	No	Two	45	14
8	No	Two	45	14
9	No	Two	45	14
10	Yes	Two	30	14
11	Yes	Two	45	14
12	Yes	Two	45	14

Table 3: Experimental design.

The subjects had complete information about both their own and everybody else's earnings table. They chose actions 1 or 2 each period. The subjects' actions were then randomly paired to determine an outcome for each pair. The subjects were informed that they were being randomly paired. Since outcomes were reported privately, subjects could not use common information about the outcomes in previous periods to coordinate on an equilibrium. Subjects confronted an anonymous participant each period.

Monetary payments were used to induce preferences. The number in the cell $\{i,j\}$ of the earnings table denotes the number of cents earned by a subject given they chose action i and the other participant they were currently paired with chose action j, see table 1. Subjects were instructed on how to derive the other participant's earnings from the earnings table.

No preplay communication of any kind was allowed. Messages were sent electronically on a PC-network.

The subjects were recruited from undergraduate economic classes at Texas A&M University in the fall of 1992. A total of 132 subjects participated in the experiment. After reading the instructions, but before the session began, the subjects filled out a questionnaire to determine that they understood how to read earnings tables. In the forty-five period sessions, which take about one and a half hours to conduct, a subject could earn as much as $18. In the seventy-five period sessions, which take about two hours to conduct, a subject could earn as much as $30.[10]

IV. EXPERIMENTAL RESULTS

The results are reported in five sections: Section *A* reports the one population no labels treatment: sessions 1 to 3; Section *B* reports the one population with labels treatment: sessions 4 to 6; Section *C* reports the two populations without labels treatment: sessions 7 to 9; Section *D* reports the two labeled populations treatment: sessions 10 to 12. Section *E* reports logistic models of individual behavior.

A. One Population no Labels.

Figure 4 reports the frequency of action 1 summed over five period intervals for the one population no labels sessions. Since there are 8 subjects per session, the frequency can range from 0 to 40. The solid horizontal line at 20 denotes the predicted equilibrium frequency, compare with figure 1. The horizontal axis measures periods.

Behavior in both the early and late periods stayed around the predicted equilibrium frequency. Although, it did drift off for short periods of time. Strictly speaking this violates the replicator dynamic and, more generally, any order compatible dynamic, see Friedman (1991). These violations are even more pronounced in the period by period population frequency data.

Deviations from the predicted frequency persist. A formal analysis of the transition between states rejects the null hypothesis that the data are

[10] The instructions are available at http://econlab10.tamu.edu/JVH_gtee/.

generated by players using the equilibrium mixed strategy of the constituent game G.[11] The persistence is asymmetric with states above the equilibrium frequency being three times as likely to persist as states below the equilibrium frequency.

With a population of eight subjects in the evolutionary game, it is possible to purify the mixed strategy equilibrium: four subjects choose 1 and four choose 2. Examination of the individual subject data reveals that the subjects fail to coordinate on this sort of a convention.

Average period earnings for the one population no labels treatment equaled \$0.20. These observed earnings coincide with predicted earnings under the mixed strategy equilibrium of the constituent game G out to the third decimal place. Hence, theory predicts earnings accurately.

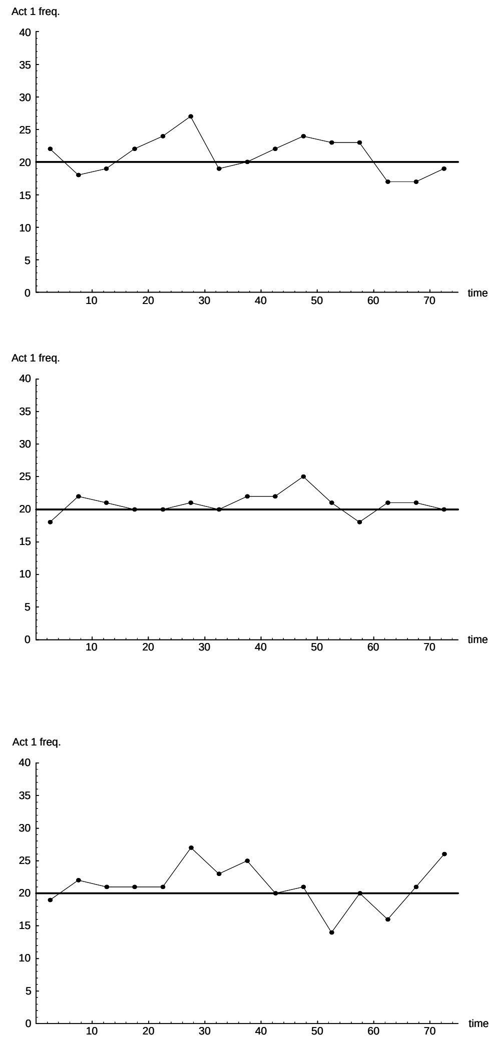

Figure 4: Sessions 1 to 3, one pop. no labels.

[11] Catagorize observations into those with action 1 fractions less than, equal to, or greater than the expected fraction of 0.5. The predicted transition matrix is {{0.13,0.101,0.13}, {0.101,0.078,0.101}, {0.13,0.101,0.13}} and the observed transition matrix was {{0.063,0.081,0.117}, {0.113,0.113,0.117}, {0.087,0.144,0.167}}. The Chi-Square statistic is 22.7, which exceeds the critical value of 14.9 at the one percent level of statistical significance. Meyers, *et al.* (1992) report similar results.

B. One Population with Labels.

Figure 5 reports the frequency of action 1 summed over five period intervals for the one population with labels sessions. Actions taken by subjects labeled row are reported on the horizontal axis and actions taken by subjects labeled column are reported on the vertical axis. Eight subjects made five choices in each five period interval. These actions are classified by label. Since there was a subject labeled row for every subject labeled column, the frequency measured on both axes can range from 0 to 20. The symmetric efficient equilibria are represented by the corners {20,0} and {0,20} in figure 5. The expected frequency under a mixed strategy equilibrium is {10,10}.

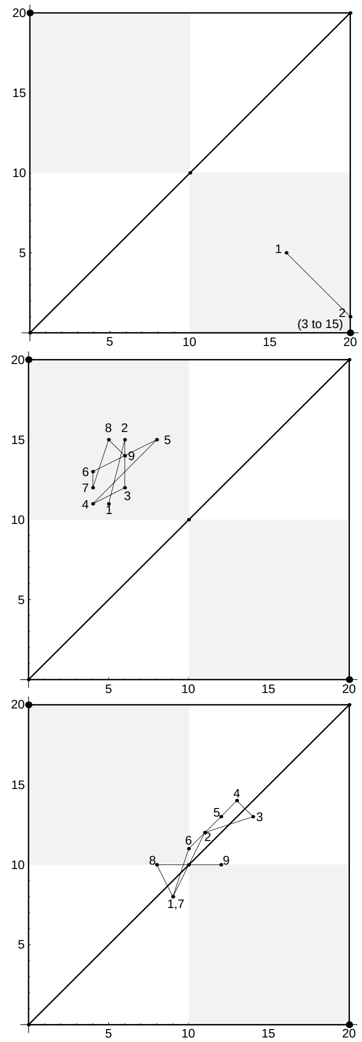

Figure 5: Sessions 4 to 6, one pop. with labels

In session 4, subjects learned to use their labels to solve their coordination problem within ten periods. After ten periods, all eight subject conform to the convention "if labeled row this period play 1 and if labeled column this period play 2." So the degree of conformity is 100 percent. While the result is a dramatic example of the origin of convention, it does not seem to be representative.[12]

[12] Session 4 certainly mislead us into cutting the number of periods from 75 in session 4 to 30 in the next session, which is labeled 10 in the text.

Session 5 wanders within the area in which all subjects have an incentive to conform to the convention “if labeled row this period play 2 and if labeled column this period play 1,” but subjects fail to coordinate on the corresponding equilibrium. For the last five periods of session 5, the degree of conformity is only 70 percent. In session 6, behavior wandered around the unstable mixed strategy equilibrium and no convention emerged, that is, subjects ignored their labels just as if they thought that labels should be strategically irrelevant.

Average period earnings in session 4 were \$0.39, which is only \$0.01 less than earnings predicted by a pure strategy equilibrium. Average period earnings in session 5 were \$0.23 Average period earnings in session 6 were \$0.20, which is equal to average earnings without labels and to earnings predicted by a mixed strategy equilibrium.

C. Two Populations without Labels.

Figure 6 reports the frequency of action 1 summed over five period intervals for the two populations no labels treatment. There were seven subjects in population 1 and seven subjects in population 2. The horizontal axis measures the frequency of action 1 in population 1 and the vertical axis measures the frequency of action 1 in population 2. These frequencies can range from 0 to 35. The efficient equilibria are represented by the points {35,0} and {0,35}. The expected frequency under the mixed strategy equilibrium is {17.5,17.5}, compare figure 3.

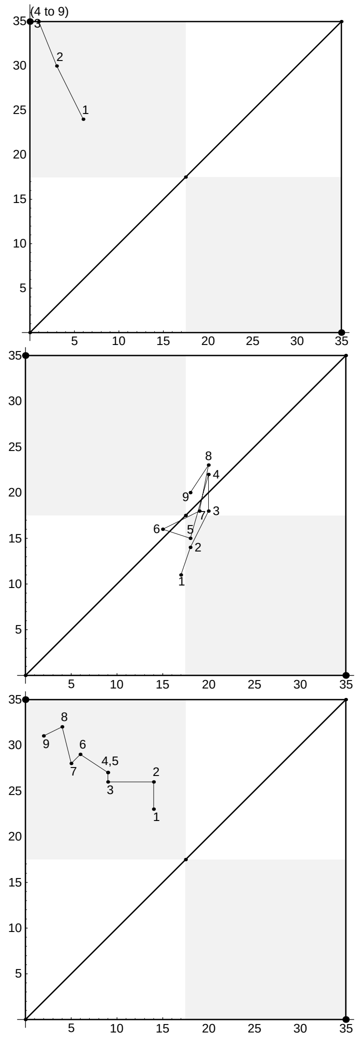

Figure 6: Sessions 7 to 9, two pop. no labels.

In session 7, subjects solved their coordination problem by adopting an action. This is possible because in a two population protocol one never meets members of one's own population. After period 15, all fourteen subjects conformed to the convention "members of population 1 always play 2 and members of population 2 always play 1." The degree of conformity is 100 percent.

In session 9, behavior co-evolved towards the same convention and it was in everyone’s interest to conform, but subjects did not coordinate on the underlying equilibrium within the 45 periods allowed. In the last five periods of session 9, the degree of conformity is 89 percent.

In session 8, behavior wandered around the unstable mixed strategy equilibrium.

Average period earnings in session 7 were \$0.37, in session 8 were \$0.17, and in session 9 were \$0.28. So the failure to coordinate on a convention costs subjects about half their earnings.

D. Two Labeled Populations.

Figure 7 reports the frequency of action 1 over five period intervals for the two labeled population sessions. The seven subjects in population 1 were labeled row and the seven subjects in population 2 were labeled column. The horizontal axis measures the frequency of action 1 in the row population and the vertical axis measures the frequency of action 1 in the column population. These frequencies can range from 0 to 35. The efficient equilibria are represented by the points {35,0} and {0,35}. The expected frequency under the mixed strategy equilibrium is {17.5,17.5}, compare figure 3.

A convention emerged in all three sessions of the two labeled populations treatment. In session 12, subjects actually coordinate on the constituent equilibrium for the last ten periods of the session. Subjects solved their coordination problem by adopting an action. In sessions 10 and 11, all subjects had a monetary incentive to conform to the convention "members of the row population play 2 and members of the column population play 1." The degree of conformity was 86 percent and 77 percent respectively. After period 35 in session 12, all fourteen subjects conformed to the convention "members of the row population play 1 and members of the column population play 2." Hence, the

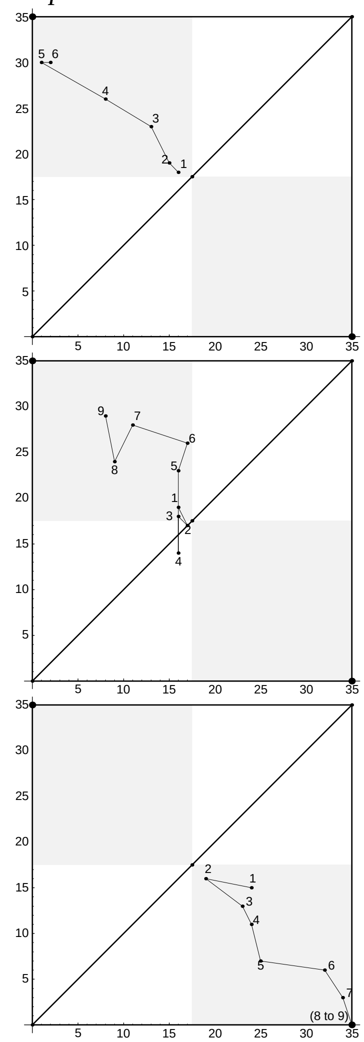

Figure 7: Sessions 10 to 12, two labeled pop.

degree of conformity was 100 percent.

Average period earnings in session 10 were \$0.25, in session 11 were \$0.22, and in session 12 were \$0.27.

The replicator dynamic provides a tractable analytical framework in which the influence of non-strategic details on behavior in an evolutionary game can be illustrated, but, while its gross predictions are fairly accurate, it makes a number of inaccurate predictions. The rate of change in a strategy is not proportional to its extant frequency as assumed by the replicator dynamic. Behavior is stochastic rather than deterministic. Moreover, as shown in the next section, behavior depends on more than the current population frequencies.

Also, it is unclear whether the mixed strategy equilibrium is really unstable as predicted by the replicator dynamic, see sessions 6 and 8. Instead, what appears to happen is that subjects get caught in the sort of correlated cycles, “cob webs”, predicted by myopic belief learning models although for short periods of time only. The population frequencies are close to the predicted frequencies but average earnings suffer during these episodes.

E. Logistic Response Models of Subject Behavior.

While the replicator dynamic provides a useful model of population dynamics, it does not provide a model of individual subject behavior. In this section, we report estimates of two belief learning models. Popular belief learning models consist of two components: a response function and an assessment rule.[13] For example, fictitious play consists of a best response function and an assessment rule based on the historical frequency of actions.

Our first empirical model of subject behavior is like fictitious play except that the best response function is replaced with the logistic response function $f(\bullet)$. Let y_{it} denote subject i's assessment of the likelyhood his opponent will play action 1 in period t. For a history of observed actions $h_{it} = (a_{i1}, a_{i2}, \ldots, a_{it-1})$, y_{it} is given by

$$y_{it} = \frac{t-1}{t} y_{it-1} + \frac{1}{t} a_{it-1}, \qquad t \geq 2,$$

where a_{it} is 1 if subject i's opponent choose action 1 in period t and zero otherwise. Let y_{i1} equal 0.5. For the one population with labels treatment, the belief variable is conditioned on the subject's label, that is, the equation is

[13] See Fudenberg and Levine (1996) for a survey of this growing literature.

iterated on the subsample of the subject's experience when labeled row and on the subsample of the subject's experience when labeled column. The value for y_{it} in the current period is that value from the subsample with the subject's current label.

The probability subject i plays action 1, p_{it}, is given by the following model of fictitious play, FP:

$$p_{it} = f(\alpha_i + \beta_i y_{it}).$$

The estimated FP model breaks the parameters α_i, β_i into a representative component α, β and an idiosyncratic component $\hat{\alpha}_i, \hat{\beta}_i$, that is, $\alpha = \alpha_i - \hat{\alpha}_i$ and $\beta = \beta_i - \hat{\beta}_i$. The model was estimated using the logistic procedure in *SAS* version 6.11. Variables were chosen for inclusion in the model using the forward selection option of the logistic procedure, which adds variables iteratively according to the score chi-square statistic until there are no variables that pass the five percent statistical significance rule of thumb.

Table 4 reports the representative component of the estimated model by treatment and for all 132 subjects combined, where *pop* denotes population and *lab* denotes labels, *std* denotes standard error, *n* denotes number of observations, *df* denotes degrees of freedom, χ^2 denotes chi-square statistic for the global hypothesis that all βs are zero, and *p-value* denotes the probability value given the chi-square statistic and degrees of freedom. The table does not report the idiosyncratic components of the fitted model, but by subtracting 2 from *df* one can deduce the number of idiosyncratic components. For example, the one population no labels fitted model has 15 idiosyncratic components.

The belief variable y_{it} is highly significant in all four treatments. Moreover, the absolute value of the point estimate increases from 6.01 for the one population no labels treatment to 11.46 for the two labeled populations treatment.

The one population no labels results contrast sharply with Mookherjee and Sopher (1994,p.82) who did not find evidence for fictitious play in an experiment with very similar best response functions. We conjecture that this difference may be due to the different matching protocols used. They used a repeated pairs protocol and we used an evolutionary matching protocol. An inertial dynamic, like fictitious play, is much more difficult to exploit in an evolutionary matching protocol than in a repeated pairs protocol.[14]

[14] This distinction between protocols that make it easy to exploit adaptive behavior and those that don't is demonstrated nicely in Bloomfield (1994).

The one population no labels fitted model is the only one that passes the Hosmer/Lemeshow goodness-of-fit (HL) test used in the logistic procedure. Cheung and Friedman (1995) report some success with belief learning models that depend on expected payoff differences between the two actions of a 2×2 game, that is, let $R(y_{it}) = \{1,-1\}.G.\{y_{it}, 1-y_{it}\}$. We estimated the following model of exponential fictious play, EFP:

$$p_{it} = f(\alpha_i + \beta_i R(y_{it})).$$

When α_i is zero, it is straight forward to show that this is Fudenberg and Levine's (1996,p.147) model of exponential fictious play.[15]

Table 5 reports the fitted EFP model. Notice that α does not differ in a statistically significant way from 0 in any treatment, and that β is highly significant in all cases. The point estimate for β increases from 0.096 for the one population no labels treatment to 0.273 for the two labeled populations treatment.

In order to compare the effect of transforming the belief variable into the expected payoff difference variable, figure 8 graphs the estimated FP and EFP models by treatment. In all four cases the EFP model is closer to the best reply function than the FP model. The figure also illustrates how moving from one population no labels to one population with labels to two populations no labels to two labeled populations twists both estimated models closer to the best response function.

The EFP model passes the HL test for all treatments, except two labeled populations. We tried several ways to get a better fit. Allowing the forward option of the logistic procedure to select between the expected payoff difference and the historically experienced payoff difference does not change the estimated model, since the historically experienced payoff difference variable is not included in the model by the procedure. (So stimulus-response models do not fit this data as well as the EFP model.) Also, the procedure when allowed to include a dummy variable for population in the model does not do so. Own and other lagged choice variables are statistically signficant. However, including own and other lagged choice variables neither changes the

[15] The EFP model differs from Cheung and Friedman's (1995,p.7) belief learning model in that it uses the logit rather than the normit function, a diffuse prior belief in the first period, and no history discounting.

point estimate for β much nor solves the lack of fit problem.[16]

If the EFP model is estimated by session rather than treatment, both sessions 11 and 12 pass the HL test. So it is session 10 that is giving the model trouble. One's initial reaction might be that session 10 is failing the HL test because it is short: only 30 periods. This is not true however. Truncating the samples at period 30 and re-estimating the EFP model by treatment has only minor effects on the parameter estimates.[17] Ironically, the *p-value* of the HL statistic rises to 0.046 when the EFP model is estimated on the two labeled population 30 period sample, which is the closest we got to passing the test for the two labeled population treatment.

[16] The estimated β drops from 0.273 to 0.212 when own and other lagged choice variables are included in the model.

[17] The estimated β's are 0.079, 0.169, 0.260, 0.252 respectively. Also, the procedure when allowed to include a period variable does not do so.

Table 4: Parameter Estimates for the FP Model

Treat. *pop lab*	α (*std*)	β (*std*)	*n*	*df*	χ^2	*p-value*
one no	2.873 (0.34)	-6.012 (0.63)	1800	17	489	0.00
one yes	4.787 (0.32)	-9.406 (0.57)	1320	18	898	0.00
two no	4.728 (0.27)	-10.081 (0.55)	1890	21	1164	0.00
two yes	5.662 (0.33)	-11.463 (0.64)	1680	24	745	0.00
all	4.615 (0.15)	-9.536 (0.28)	6690	71	3270	0.00

Table 5: Parameter Estimates for the EFP Model

Treat. *pop lab*	α (*std*)	β (*std*)	*n*	*df*	χ^2	*p-value*
one no	-0.115 (0.077)	0.096 (0.0089)	1800	18	519	0.00
one yes	0.143 (0.11)	0.172 (0.015)	1320	20	956	0.00
two no	-0.044 (0.10)	0.249 (0.025)	1890	30	1247	0.00
two yes	-0.084 (0.090)	0.273 (0.020)	1680	35	852	0.00
all	-0.003 (0.047)	0.214 (0.0089)	6690	104	3580	0.00

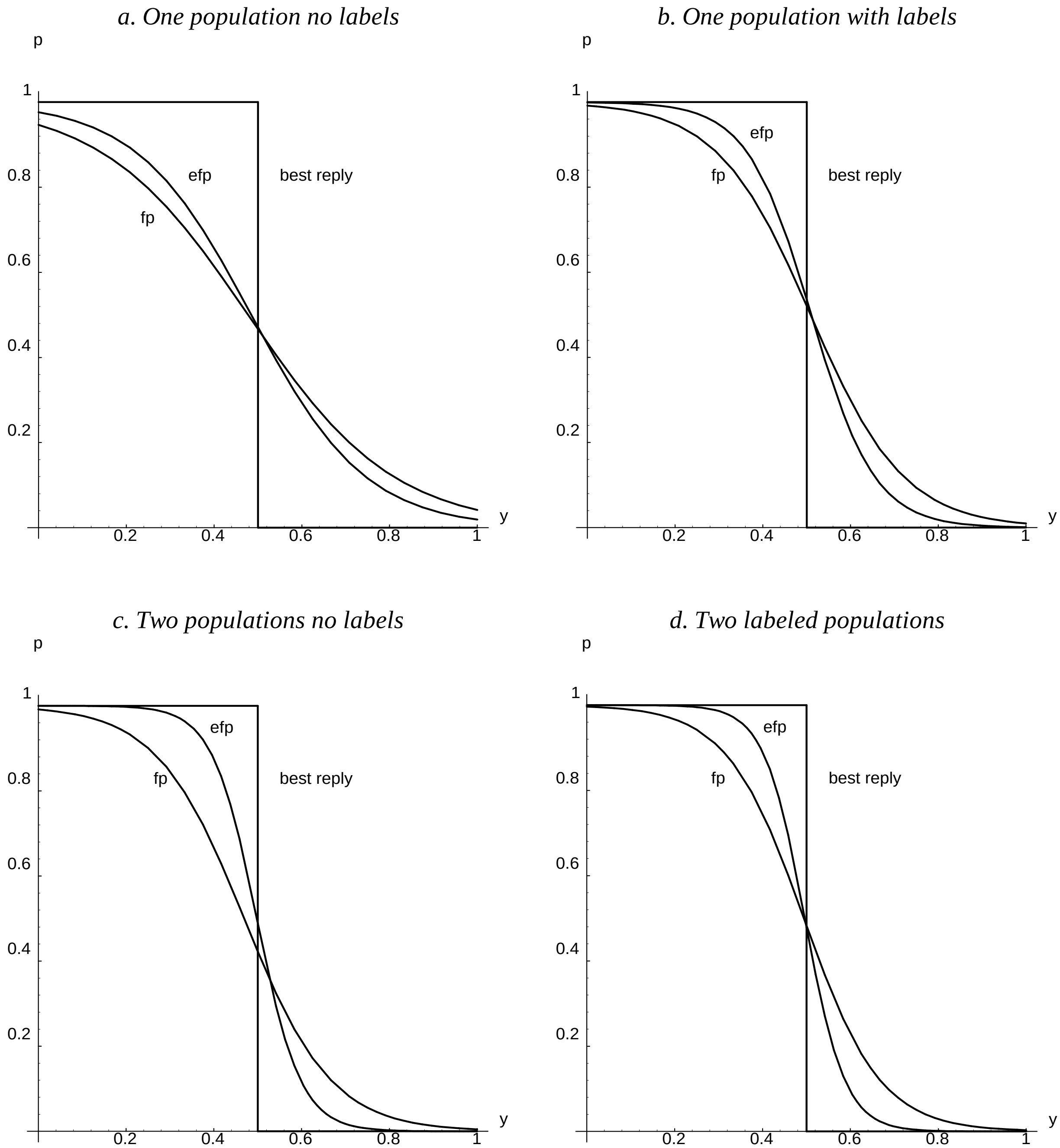

Figure 8: Estimated fictitious play (fp) and exponential fictitious play (efp) models.

V. SUMMARY AND CONCLUSION

Table 6 summarizes the results. Without non-strategic details, the outcome was inefficient as predicted by symmetry. Labeling subjects row or column but holding the number of populations constant at one allowed two of three sessions to coordinate on a convention, which improved efficiency. Two of three sessions under the two population no labels treatment coordinated on a convention, which also improved efficiency. A convention emerged in all three sessions of the two labeled populations treatment. It is in this sense that it was the most effective treatment. The experiment demonstrates that it is possible for conventions based on labels or on populations to emerge.

We were pleasantly surprised by the ability of the exponential fictitious play model to fit our individual subject data. The experiment was designed and conducted before Fudenberg and Krep's (1993) published their seminal work on smooth fictitious play. Yet, of the numerous empirical models we have tried, this is the first which seems close to being right to us. We will certainly want to use it in designing future experiments.

Ses	First five periods.	Predict. Conv.	Last five periods.	Actual Conv.	Deg. of conf.	Average Period Earnings
One Pop/No Labels						
1	{22,18}	None	{19,21}	None	--	$0.19
2	{18,22}	None	{20,20}	None	--	$0.22
3	{19,21}	None	{26,14}	None	--	$0.20
One Pop/Labels						
4	{16,4,5,15}	$\{e_1,e_2\}$	{20,0,0,20}	$\{e_1,e_2\}$	100	$0.39
5	{5,15,11,9}	$\{e_2,e_1\}$	{6,14,14,6}	$\{e_2,e_1\}$	70	$0.23
6	{9,11,8,12}	$\{e_1,e_2\}$	{12,8,10,10}	None	--	$0.20
Two Pop/No Labels						
7	{6,29,24,11}	$\{e_2,e_1\}$	{0,35,35,0}	$\{e_2,e_1\}$	100	$0.37
8	{17,18,11,24}	$\{e_1,e_2\}$	{18,17,20,15}	None	--	$0.17
9	{14,21,23,12}	$\{e_2,e_1\}$	{2,33,31,4}	$\{e_2,e_1\}$	89	$0.28
Two Pop/Labels						
10	{16,19,18,17}	$\{e_2,e_1\}$	{2,33,30,5}	$\{e_2,e_1\}$	86	$0.25
11	{16,19,19,16}	$\{e_2,e_1\}$	{8,27,29,6}	$\{e_2,e_1\}$	77	$0.22
12	{24,11,15,20}	$\{e_1,e_2\}$	{35,0,0,35}	$\{e_1,e_2\}$	100	$0.27

Table 4: Summary table. Ses denotes session. Pop. denotes population. Conv. denotes convention. The vectors report either the frequency of action 1 and 2 or the freqeuncy of action 1 and 2 by label or population or both. Deg. of Conf. denotes degree of conformity.

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